

Diodorus's Master Argument

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In my “Remarks on Stoic Logic” that I wrote for you last year, I mentioned Diodorus Cronus’s trilemma, or master argument. This is the argument that Diodorus gave in defence of his threefold claim:

1. what is possible is what is or will be the case,
2. what is necessary is what is and always will be the case,

and

3. what is impossible is what neither is nor ever will be the case.

Unfortunately, like so much of what was written in ancient times, we do not know the details of Diodorus’s argument. We do know, as I mentioned in my “Remarks,” that both Chrysippus, who first developed stoic logic, and Diodorus’s pupil, Philo, disagreed with Diodorus about the meaning of possibility and necessity. But even though they disagreed with Diodorus’s conclusion, they did think that his argument for that conclusion was valid.

A valid argument is not the same as a sound argument, however. An argument is *valid* if it is impossible to have the premises true and the conclusion false. A *sound* argument is both valid and has true premises, which means that the conclusion of a sound argument must also be true. Chrysippus and Philo said that the conclusion of Diodorus’s argument was not true even though they thought that the argument was valid. That means that they had to reject one or more of his premises, or assumptions. In this note I would like to discuss these premises and a possible reconstruction of Diodorus’s argument.

1 Tensed Propositions

Many philosophers think that propositions are either eternally true or eternally false. This may be because they think that propositions are abstract entities like numbers and do not exist in time. Of course a statement such as $2 + 3 = 5$ is not just sometimes true but always true.

The stoics did not believe in abstract entities, but they were unsure about the nature of propositions (*lekta*). Nevertheless, whether propositions are abstract entities or not, for Diodorus propositions are tensed so that a proposition, such as is expressed by the sentence “The pope is Italian”, might be true in the past but not true now in the present, and that a proposition such as “George W. Bush is president is of the USA” is true now but will be false in the future.

As noted in my “Remarks” we can represent the tensed aspect of a proposition by means of tense operators, such as P and F , which we can read as:

P : It was the case that ...

and

F : It will be the case that ...

We assume that the present tense is represented by the present tense of the verb or predicate of a sentence. Then, just as we have the following reading:

$\neg Fp$: It will never be the case the case that p

so too we have the following:

$\neg F\neg p$: It will always be the case that p

and

$\neg P\neg p$: It was always be the case that p .

Diodorus's claim that a proposition p is possible if, and only if, it is either true or will be true can then be represented as:

$$\diamond p \leftrightarrow p \vee Fp,$$

where $\diamond p$ is read as "It is possible that p ". Similarly, Diodorus's claims that a proposition p is necessary if, and only if, p is true and always will be true hereafter can be represented by:

$$\Box p \leftrightarrow p \wedge \neg F\neg p,$$

where $\Box p$ is read as "it is necessary that p ". His claim that p is impossible if, and only if, p is false and never will be true hereafter can be represented by:

$$\neg\diamond p \leftrightarrow \neg p \wedge \neg Fp.$$

These three claims are really equivalent to one another. The two about possibility and impossibility are clearly equivalent given that the negation of a disjunction is equivalent to a conjunction of negations. The claim about necessity follows from the duality of necessity and possibility, i.e., that

$$\Box p \leftrightarrow \neg\diamond\neg p.$$

I will hereafter not distinguish between the three claims but just discuss Diodorus's claim for possibility, i.e., his claim that what is possible is what is or will be the case:

$$\diamond p \leftrightarrow p \vee Fp.$$

This is the claim for which Diodorus is most remembered. It is an important claim because it purports to tell us how the concept of possibility (and necessity) is to be understood in terms of time, i.e., in terms of our talk about the past, the present and future. We should also keep in mind here that in addition to a discussion of possibility and necessity, the Stoics were the first to develop classical truth-functional propositional logic, which we will assume hereafter.

2 The Unalterability of the Past

In the few commentaries on Diodorus Cronus's trilemma, there are two main premises. These are:

- (1) An impossible proposition never follows from a possible one.
- (2) Every true proposition about the past is necessary.

The denial of Diodorus's claim about possibility is the third part of Diodorus's trilemma. This denial is:

- (3) There is a proposition that is possible, but which is not true now nor ever will be true (in the future).

A trilemma is a collection of three propositions that are said to be in conflict and cannot all be taken as true. (A dilemma consists of two propositions that are said to be in conflict and cannot both be accepted as true.)

Diodorus said that the premises (1) and (2) must be taken as true and therefore that (3) must be false, which is Diodorus's thesis about possibility, namely:

A proposition is possible if, and only if, it is either true (now) or will be true.

The question is: are (1) and (2) true? And does (1) and (2) really imply that (3) is false, and hence that possibility means what Diodorus said it means? What was Diodorus's argument for why (1) and (2) imply (3)?

Let us consider premise (1) first. This is the statement that an impossible proposition cannot follow from a possible proposition. It certainly seems to be true and unproblematic. An equivalent way of stating this is that if p follows from q , i.e., if q implies p , then q is possible only if p is possible. In modal terms we can represent the statement that q implies p as the necessity of the conditional ($q \rightarrow p$), i.e., as $\Box(q \rightarrow p)$. Then premise (1) is equivalent to the following modal thesis:

$$\Box(q \rightarrow p) \rightarrow (\Diamond q \rightarrow \Diamond p), \tag{A}$$

This thesis, (A), is equivalent to the modal law of the distribution of \Box over \rightarrow , i.e.,

$$\Box(q \rightarrow p) \rightarrow (\Box q \rightarrow \Box p),$$

and this law is a basic law of every modal logic. It is, in other words the most minimal statement you can make about necessity and possibility. (Can you see why these last two formulas are equivalent? Note that we can substitute complex sentences for p and q , e.g., substitution of $\neg p$ for p and $\neg q$ for q will show you why the two formulas are equivalent theses.)

What the sentence (A) says is that if q implies p , then q is possible only if p is possible. Can you see why this is a modal way of representing premise (1)? Stated in this way we can clearly accept premise (1).

What about premise (2)? Do you think it is true? Doesn't it say that we cannot change the past, and therefore a true proposition about the past

is necessary. What about time travel? Can we someday travel in time and change the past? Can God change the past? Aristotle did not think so. In his *Nichmachean Ethics*, Aristotle says that “what is done cannot be undone” and that Agathon is right when he says “even God is deprived the power of making what has been done not to have happened” (Book 6, chapter 2). God could have made another world in which the past was different, but could God change the past of our world and it still be our world?

Note that if a proposition p was true, i.e., if it was the case that p , then Pp is now true and will henceforth always be true, i.e., $\neg F\neg Pp$, and hence, by Diodorus’s notion of necessity, Pp is necessary, i.e., $\Box Pp$ is true. Premise (2) is true, in other words, if Diodorus is correct about the meaning of necessity.

But that begs the question. We cannot prove that Diodorus’s claim about possibility is correct by assuming his claim as a way to justify one of the premises of his argument. So how otherwise can we justify premise (2)? Well, we will not try to justify premise (2) here, but for the purpose of discussion just allow that it is a necessary truth. That is, we will take the thesis that if a proposition was the case, then it is necessary that it was the case as a valid thesis:

$$Pp \rightarrow \Box Pp. \tag{B}$$

Note that in assuming that (B) is a valid thesis we are allowing that it is therefore necessary, i.e., that

$$\Box(Pp \rightarrow \Box Pp)$$

is also a valid thesis by the modal logic rule of necessitation. This the rule that if a statement is valid, then it is necessary. The two premises, or assumptions, of Diodorus’s argument are accordingly (A) for premise (1) and (B) for premise (2).

3 Reconstructing Diodorus’s Argument

The philosopher and historian Edward Zeller thought that Diodorus’s argument was a *reductio ad absurdum* and that it went something like this. Suppose there is a proposition p that is possible but not now true nor will it ever be true in the future, i.e., suppose

$$\Diamond p \wedge \neg p \wedge \neg Fp.$$

Then in the future, when the present becomes the past, $\neg p$ will become necessary by premise (2). But if $\neg p$ becomes necessary, then p becomes impossible, which means, contrary to premise (1) that an impossible proposition follows from a possible proposition.

Zeller’s argument does not work, however. Apparently, Zeller has $\Box\neg p$ in mind when $\neg p$ becomes necessary, and hence p impossible, i.e., $\neg\Diamond p$, which is the negation of the assumption $\Diamond p$; and of course $\neg\Diamond p$ does not follow from $\Diamond p$. But the proper representation of the claim that the negative proposition $\neg p$,

which true now, will (in the future) become necessary (because then it will be about the past) is $\Box FP\neg p$, i.e., that what is necessary is that it will be that it was the case that $\neg p$, and from this we do not have $\neg\Diamond p$.

There is another reconstruction of Diodorus's argument that my old friend Arthur Prior (now deceased) once gave. It too doesn't work, as Prior came to realize. But the reconstruction and the reason it doesn't work is interesting.

What Prior noted was that if

$$\neg p \wedge \neg Fp \rightarrow \neg\Diamond p$$

is provable, then (by contraposition and truth-functional logic) so is

$$\Diamond p \rightarrow p \vee Fp.$$

This is all we need because the converse claim that if p is or will be true, then p is possible, i.e.,

$$p \vee Fp \rightarrow \Diamond p$$

is certainly a valid thesis. In fact, the statement that if p is true then it is possible, i.e., $(p \rightarrow \Diamond p)$, is a basic modal law. It is clear that the statement that if p will be true, then it must be possible is certainly acceptable as well. So we need consider and try to prove only the one direction, namely

$$\Diamond p \rightarrow p \vee Fp.$$

In addition to the theses (A) and (B), Prior added two other theses that he took as valid for tense logic. The first is

$$p \rightarrow \neg P\neg Fp,$$

which says that if p is now true, then it always was the case (in the past) that it was going to be true (in the then future). This is certainly a valid thesis of tense logic, and therefore it too can be taken as necessary, i.e., by the modal logic rule of necessitation that a valid thesis is necessary,

$$\Box(p \rightarrow \neg P\neg Fp) \tag{C}$$

is a valid thesis as well.

Now note that

$$\Box(p \rightarrow \neg P\neg Fp) \rightarrow (\Diamond p \rightarrow \Diamond\neg P\neg Fp)$$

is an instance of thesis (A), from which, by (C) and the rule of modus ponens, we therefore have

$$\Diamond p \rightarrow \Diamond\neg P\neg Fp.$$

Accordingly, by contraposition (truth-functional logic),

$$\neg\Diamond\neg P\neg Fp \rightarrow \neg\Diamond p$$

is also valid. But this formula, by the duality of necessity and possibility, is equivalent to

$$\Box P\neg Fp \rightarrow \neg\Diamond p.$$

Finally, to complete the proof, there was one last assumption that Prior made about tense logic. This was the thesis that if a proposition p is now false and will henceforth always be false, then it was the case that p will not be true thereafter, in symbols:

$$\neg p \wedge \neg Fp \rightarrow P\neg Fp. \tag{D}$$

But by thesis (B) that the past is unalterable, the formula

$$P\neg Fp \rightarrow \Box P\neg Fp$$

then follows, and from this formula and (D) we have

$$\neg p \wedge \neg Fp \rightarrow \neg\Diamond p,$$

which, by contraposition and truth-functional logic, is equivalent to

$$\Diamond p \rightarrow p \vee Fp,$$

which, as we said, is all that we needed to complete Diodorus's argument about possibility.(QED)

4 Is Time Discrete or Continuous

The above reconstruction of Diodorus's master argument is flawed in only way, namely thesis (D). What's wrong with (D)? Well, nothing if time is discrete, i.e., if after each moment there is a next moment with no moments in between. The integers are discrete: each integer is followed (and preceded) by just one integer with no other integer between them. The rational numbers are not discrete but dense, on the other hand, because between any two rational numbers x and y there is another rational number, namely x/y . The rational numbers are not continuous, however. One of Pythagorus's students was said to have shown this when he proved that the square root of two, $\sqrt{2}$, is not a rational number. Can you see why this is so? (It was rumored that this result so upset the Pythagoreans that they murdered the student and tried to keep the result a secret. But you don't need to worry about that in Professor Addona's class.)

Can you see why (D), taken as a valid thesis for all propositions p , assumes that time is discrete? Suppose, e.g., that p is false at a moment t and false thereafter, i.e., at t , the conjunction $\neg p \wedge \neg Fp$ is true. If time is discrete (and t is not the first moment of time), then there is a moment t' that immediately precedes t , and therefore at t' it is true that p will never be true thereafter, i.e., $\neg Fp$ is true at t' . This shows us why (D), the conditional

$$\neg p \wedge \neg Fp \rightarrow P\neg Fp,$$

is valid if time is discrete.

If time is not discrete, however, but continuous, then between t and any moment preceding it there are infinitely many other moments at any one of which p might true, in which case even though the conjunction $p \wedge \neg Fp$ is true at t , the consequent of (D), namely, $P\neg Fp$ need not also be true at t , in which case the conditional will be false at t . This shows us that (D) is not a valid thesis if time is not discrete but continuous. In that case the above reconstruction of Diodorus's master argument is valid but not sound because one of its assumptions is not a valid thesis unless time is discrete.

So the question now is: is time discrete? Indeed, *can* time be discrete? Doesn't Zeno's Stadium argument show that time cannot be discrete if motion is possible, which Parmenides, Zeno's teacher said was impossible. How is motion possible? Does it require that space and time be continuous? That is what many philosophers say.

But because space and time are "quantized" in quantum physics, there is a smallest physically possible length — namely, the "Planck length" of 10^{-33} cm. — and a smallest physically possible time, namely, the time it takes for light to cross the Planck length, which is 10^{-43} seconds. Doesn't this mean that space and time are not infinitely divisible and hence not continuous?

Finally, is there any other way to reconstruct Diodorus's master argument? I do not know, but it is something to think about.